Normal mode analysis

Expand about a minimum:

$$V(x) = V_0 + \frac{1}{2} k (x - x_0)^2 + \ldots$$

$$m \frac{d^2 x}{dt^2} = -\frac{\partial V}{\partial x} = -k(x - x_0) \Rightarrow -kx$$

Try a periodic function: $x(t) = A\cos(\omega t)$

$$-A\omega^2 \cos(\omega t) = -A \frac{k}{m} \cos(\omega t)$$

$$\Rightarrow \quad \omega = (k/m)^{1/2}$$

In multiple dimensions:

$$-\underset{\approx}{K}\,\underset{\sim}{x} = \underset{\approx}{M}\,\underset{\sim}{\ddot{x}} \quad \Rightarrow \quad \text{try} \quad \underset{\sim}{x} = \underset{\sim}{q} \cos(\omega t)$$

$$-\underset{\approx}{K}\,\underset{\sim}{q}\,\cancel{\cos(\omega t)} = -\omega^2 \underset{\approx}{M}\,\underset{\sim}{q}\,\cancel{\cos(\omega t)}$$

$$\underset{\approx}{K}\,\underset{\approx}{M}^{-1/2}\left(\underset{\approx}{M}^{1/2}\underset{\sim}{q}\right) = -\omega^2 \underset{\approx}{M}^{1/2}\left(\underset{\approx}{M}^{1/2}\underset{\sim}{q}\right)$$

$$\underset{\approx}{M}^{-1/2}\,\underset{\approx}{K}\,\underset{\approx}{M}^{-1/2}\,\underset{\sim}{q}' = -\omega^2 \underset{\sim}{q}' \quad \Rightarrow \quad \underset{\approx}{K}'\,\underset{\sim}{q}' = -\omega^2 \underset{\sim}{q}'$$